Sree Kerala Varma College, Thrissur

Functional Analysis I

Assignment 2

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1. Let $1 \leq p < r \leq \infty$. Show that $\ell^p \subsetneq \ell^r$.

2. Let E be a measurable subset of $\mathbb{R}$ with $m(E) < \infty$ and let $1 \leq p < r \leq \infty$. Show that $L^r(E) \subsetneq L^p(E)$. If $m(E) = \infty$, then show that neither $L^p(E) \subseteq L^r(E)$ nor $L^r(E) \subseteq L^p(E)$.

3. Show that ℓ^p is a complete metric space; $1 \leq p \leq \infty$.

4. Show that L^p is a complete metric space; $1 \leq p \leq \infty$.

5. Show that the metric space ℓ^p is separable for $1 \leq p < \infty$ while ℓ^∞ is not.

5. Show that the metric space L^p is separable for $1 \leq p < \infty$ while L^∞ is not.