Sree Kerala Varma College, Thrissur

Functional Analysis I

Assignment 1

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1 A metric space X is said to be **complete** if every Cauchy sequence in X is convergent in X. For $1 \leq p \leq \infty$ and $n = 1, 2, \cdots$, show that the metric space $(\mathbb{K}^n, d_p)$ is complete where $d_p(x, y) = \|x - y\|_p$. Give an example of a metric space which is not complete.

2 A metric space is said to be **separable** if it has countable dense subset. For $1 \leq p \leq \infty$ and $n = 1, 2, \cdots$, show that the metric space $(\mathbb{K}^n, d_p)$ is separable.

3 For $x \in \mathbb{K}^n$, show that $\|x\|_\infty \leq \|x\|_2 \leq \|x\|_1$ and $\|x\|_1 \leq \sqrt{n}\|x\|_2 \leq n\|x\|_\infty$.